Research on Decision Optimization and Sampling Inspection Schemes in the Production Process

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Abstract. In the production process, decision-making problems usually involve multiple stages, and the decision-making of each stage often affects the efficiency, cost and profit of the whole production chain. This article mainly studies the decision-making of the production process. For samples with a nominal substandard rate of 10%, small and large samples were analyzed by sequential sampling, and the results were tested using the ASN efficiency index. The acceptance rate and rejection rate obtained were inferred whether this batch of spare parts was accepted. During the production process, 10,000 pieces of data are randomly generated for decisions that require parts inspection and simulated under different circumstances. Take 80% of the 10,000 data as a training set and 20% as a test set, calculate the expectations of each decision point, and find out the best procurement and production strategies, so as to derive the optimal method. For m processes, n spare parts, divide the problems into sub-problems, use dynamic planning to gradually find out the optimal decision-making of each problem, and use the DP matrix to solve the optimal result. This method can not only effectively reduce the number of sampling tests and reduce the cost of testing, but also optimize the production process through dynamic planning and improve the efficiency of resource utilization. The research results provide enterprises with operable theoretical basis and practical guidance, which helps enterprises to reduce costs and increase efficiency in the fierce market competition and improve overall economic benefits.

Keywords: Production decision, decision tree, sequential sampling inspection, parts inspection.

1. Introduction

In today's industrial setting, production decisions aim to use resources efficiently for the best economic outcomes. Traditional methods are slow and error-prone. Sequential sampling techniques offer a better approach. Toninato et al. (2024) showed its usefulness in decision-making [1]. Overmeire et al. (2023) used it for environmental monitoring [2]. Insights from sequential sampling models: Sequential sampling models are computational models used to study decision-making processes. They reveal human decision-making mechanisms by simulating how people process information step by step when making decisions [3]. Shim and Kim (2024) applied it in quality control with advanced analytics [4]. Sahinoglu and Capar (2022) optimized error rates in quality control [5]. Amir Salar et al. (2019) developed a sustainable supply chain model [6]. Skre et al. (2023) improved manufacturing processes [7]. Kim et al. (2023) assessed safety sampling plans [8], and Zimmermann and Brandtner (2024) enhanced supply chain management with machine learning [9]. Ma, Z. & Wang, Y. (2024). Leveraging artificial intelligence and machine learning technologies to enhance the quality control efficiency of electric vehicle production lines [10]. These advancements show that integrating simulation evaluation and machine learning can significantly improve decision quality and efficiency. In short, modern techniques like sequential sampling and machine learning provide powerful tools for making better production decisions, enhancing accuracy and efficiency while reducing risks.

2. Research on Component Inspection Issues

2.1. Sequential sampling inspection

In this paper, the design of the inspection scheme aims to minimize the number of inspections. The paper deals with the issue of component inspection, where the sampled inspection data can be approximated as following a binomial distribution since the inspection result of each component is a binary event, with only two possible outcomes: qualified or unqualified. The data follows a binomial distribution, which can be expressed as:

$$X \sim Bin(n, p) \tag{1}$$

Among them, n is the number of independent tests, p can be the rate of secondary products, and X is the quantity of detected secondary products. If the sample size is large enough, based on the central limit theorem: regardless of the shape of the original distribution, as long as you repeat the number of independent tests.

When n is large enough, the mean distribution of the sample will tend to a normal distribution. Two-tailed distribution can approximate to normal distribution. Therefore, in this case of large sample size, we can use the normal distribution to perform approximate calculations to design and analyze sampling inspection schemes. That is:

$$X \sim N(\mu,\sigma^2) \tag{2}$$

Among them, X is a random variable that represents data obeying the normal distribution. μ is the mean value of the normal distribution, representing the central position of the distribution. σ is the variance of the normal distribution, indicating the dispersion degree of the data, and the square root of the variance σ is the standard deviation.

2.2. Determine Data Distribution

This paper tries to reduce the number of products to be inspected as much as possible under the premise of basically the same sampling effect. In sequential sampling inspection (SPRT), the number of samples is not determined in advance, but adjusted dynamically according to the results of each inspection. The key to stopping sampling is whether the sample rate can meet specific decision criteria. Therefore, the sequential ratio probability test is used to judge the parts, and the number of inspections and the acceptance and rejection of parts are required.

First, define the efficiency indicator ASN:

$$ASN = \frac{\ln(\frac{A}{B})}{\ln(\frac{p_1}{p_0})} \tag{3}$$

Where p_1 represents the defect rate under H_1, and p_0 represents the defect rate under H_0. The constants A and B are defined as:

$$A = \frac{1}{(1-\beta)^{\frac{1}{\alpha}}}, B = \frac{(1-\alpha)}{1-(1-\alpha)^{\frac{1}{\beta}}} \tag{4}$$

2.3. Specific Results for Different Reliabilities

In the above research, it is necessary to set parameters. In order to better select spare parts, it is assumed that p_0 is 0.15, the defect rate is p, and the overall assumption is $H_0{:}p \le p_0; H_1{:}p \ge p_1$

Set the number of simulations to 10,000 times, record the sample size for each simulation, and count the acceptance and rejection situations under different assumptions. The formula for calculating the ASN is:

$$\mathrm{ASN} = \frac{(1-\beta)\log\left(\frac{1-\beta}{\alpha}\right) + \beta\log\left(\frac{\beta}{1-\alpha}\right)}{(p_1 - p_0)^{2}} \tag{5}$$

Assuming the simulation draws a genuine product, the contribution rate is:

$$\log\left(\frac{1-p_0}{1-p_1}\right) \tag{6}$$

The log-likelihood ratio is:

$$\mathrm{LLR} = \sum\left[\log\left(\frac{p_0}{p_1}\right)\cdot \mathrm{ob} + \log\left(\frac{1-p_0}{1-p_1}\right)\cdot\,(1-\mathrm{ob})\right] \tag{7}$$

Where ob represents the data of the simulation sample, it is assumed that the simulated data can only be 0 or 1.0 indicates a qualified product, and 1 indicates an unqualified product.

Finally, each simulation generates different observation data, calculates the corresponding counterfactual ratio, and compares it with $\log(A)$ and $\log(B)$ Determine whether to reject, accept or continue based on the size of the comparison. Discuss the problem according to the situation:

(1) At a 95% confidence level, the values of α and β are 0.05 and 0.1 respectively. By calculation, as shown in Table 1, the number of rejections and acceptances is:

Table 1. 95% Confidence Simulation Scenario

Sample Size	Number of Acceptances	Number of Remaining Times	Number of Rejections	Total Number of Simulations
20	0	1000	0	0
40	7	9993	0	7
60	26	9830	144	170
80	14	9622	364	378
80	51	8812	1137	1188

When the sample size is 80~100, the rejection rate increases, and the rejection rate also increases. Therefore, reject H_0, that is, $p \geq p_1$, indicating that this null hypothesis the quality of the parts is less than p_0, choose to buy this batch of parts

(2) At a 90% confidence level, the α and β are: 0.1, 0.1. Through calculation, as shown in Table 2, the number of rejections and acceptances is:

Table 2. 90% Confidence Simulation Scenario

Sample Size	Number of Acceptances	Number of Remaining Times	Number of Rejections	Total Number of Simulations
20	5	9995	0	0
40	40	9792	168	208
60	41	9450	509	550
80	110	8101	1789	1899
80	106	7799	2095	2201

Rejection indicates the rejection of the reserve hypothesis, $p < p_1$, indicating that the defect rate of the spare parts is less than p_0. Therefore, under both circumstances, choose to purchase.

3. Decision Tree Model in Production Decision Inspection and Disassembly

3.1. Construction of Decision Tree Model

Decisions need to be made for each production link, and six different situations are faced. To do this, it is necessary to reduce costs as much as possible while striving to increase revenue. Since sales volume has a direct impact on revenue and cost, sales volume must be considered when solving this problem. Therefore, set the sales volume of spare part 1 to $s1$, and the sales volume of spare part 2 to $s2$.

Establish objective function:

$$G_{max} = R - C \tag{8}$$

Among them, the revenue is C, and the inspection cost is C. In this question, you need to choose whether to inspect or not, disassemble or not, etc. In order to facilitate subsequent problem solving, set up binary variables: $E(0)(1)$: indicates whether to inspect spare parts, $E(0)(2)$: indicates whether to inspect finished products, $E(0)(3)$ indicates whether to disassemble unqualified finished products, $E(0)$ indicates no action. According to the related formula in the production process, $E0$ is the raw material cost:

$$E0 = n1 \cdot q1 + n2 \cdot q2 \tag{9}$$

Where $n1$, $n2$ are the usage quantities of parts 1 and 2 respectively, and $q1$, $q2$ are the purchase unit prices of parts 1 and 2. $E1$ is the inspection cost of accessories:

$$E1 = n_1 \cdot c_{d1} + n_2 \cdot c_{d2} + n_1 \cdot r_1 \cdot p_1 + n_2 \cdot r_2 \cdot p_2 \tag{10}$$

Where c_{d1}, c_{d2} are the testing costs of parts 1 and 2 respectively, r_1, r_2: are the second-grade rates of parts 1 and 2 respectively, p_1, p_2 are the purchase unit prices of parts 1 and 2 respectively. $E2$: Assembly and testing cost of finished products:

$$E2 = N \cdot (c_a + c_d) + N \cdot s \cdot \left(1 - r_f - \frac{E(0)(1) \cdot (n_1 \cdot r_1 + n_2 \cdot r_2)}{N}\right) \tag{11}$$

Among them, s is the market selling price of each finished product, and r_f is the secondary rate of the finished product. $E3$: The cost of dismantling defective products:

$$\begin{gathered} E3 = (N \cdot r_f + E(0)(1) \cdot (n_1 \cdot r_1 + n_2 \cdot r_2)) \cdot c_r - (N \cdot r_f) \cdot (p_1 + p_2) \\ - E(0)(1) \cdot (n_1 \cdot r_1 \cdot p_1 + n_2 \cdot r_2 \cdot p_2) \end{gathered} \tag{12}$$

Among them, c_r is the disassembly cost of finished products. $E4$: Exchange loss:

$$E4 = (E(0)(1) \cdot (n_1 \cdot r_1 + n_2 \cdot r_2) + |E(0)(2) - 1| \cdot N \cdot r_f) \cdot l \cdot E(0)(3) \tag{13}$$

From the above formula, we can obtain the formula for maximizing the final profit as:

$$G_{max} = N \cdot s - [E_0 + E(0)(1) \cdot E_1 + E(0)(2) \cdot E_2 + E(0)(3) \cdot E_3 + E_4] \tag{14}$$

By constructing a decision tree and analyzing the expected value of different decision paths, choose the path with the smallest cost as the optimal decision.

3.2. Model Solving

After establishing the decision tree in this article, 10,000 data are randomly generated, 80% as training set, 20% as simulation data. The maximum and minimum values of each parameter are respectively set to meet the data of the problem, ensuring that the randomness of the data within the range given by the topic. In this topic, the sample volume is unknown, when the sales volume sample is too large, will it affect the decision-making, thereby affecting the decision-making. Through different situations of testing respectively, we obtain the relationship between the number of finished products N and the profit with spare parts 1 and spare parts 2:

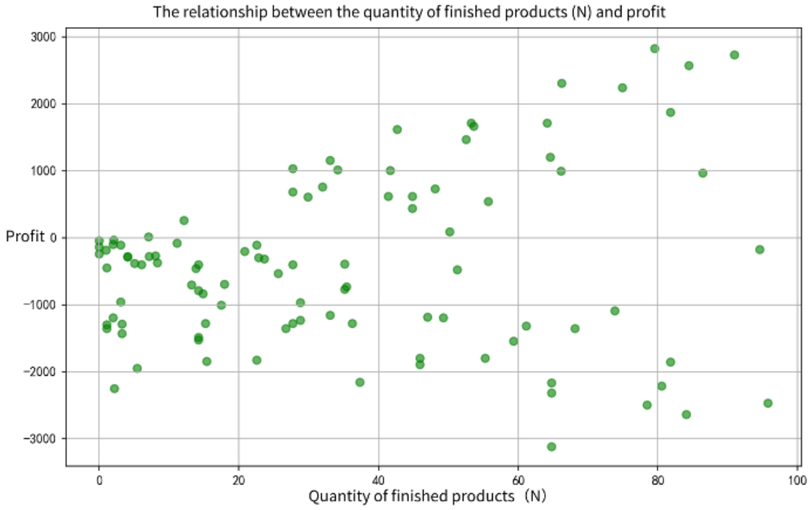

Figure 1. Relationship between Finished Product Quantity N and Profit

As shown in Figure 1, an increase in the number of finished products does not necessarily lead to stable profit growth. In some places with higher numbers of finished products, there is a significant fluctuation in profits. Decision-making may have a negative impact on profitability.

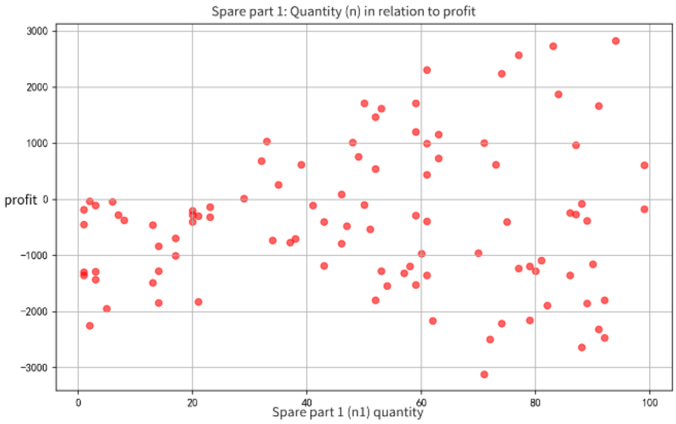

Figure 2. Relationship between Quantity of Part 1 and Profit

Figure 3. Relationship between Quantity of Part 2 and Profit

Analyzing Figures 2 and 3, the paper observe that they are similar to Figure 1 in that an increase in quantity does not necessarily lead to a corresponding increase in profit. Therefore, the change in profit is not merely due to changes in quantity. First, solve for stage (1) (2) and set the strategy as shown in Table 3:

Table 3. Decision Strategy for Phases (1) and (2)

Strategy Code	Inspection of Part 1	Inspection of Part 2	Inspection Cost	Disassembly Cost
Strategy 1	Yes	Yes	No	No
Strategy 2	Yes	No	No	Yes
Strategy 3	No	No	No	No
Strategy 4	No	No	Yes	Yes

How to make more accurate decisions by introducing different numbers and strategies of spare parts, set the number of spare parts as 20000. Draw the total cost chart under the strategy of different quantities of spare parts:

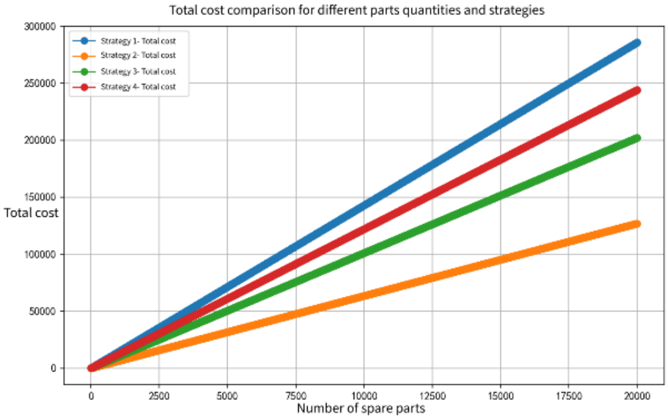

Figure 4. Total Cost under Different Parts Quantities and Decisions

Consider six different situations:

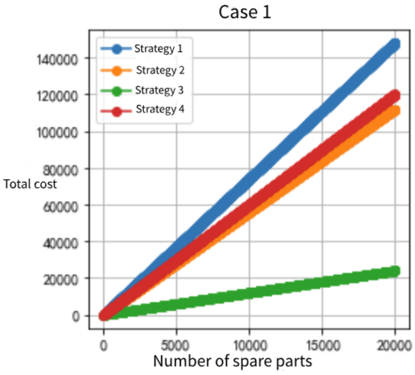

Figure 5. Total Cost for Scenario 1

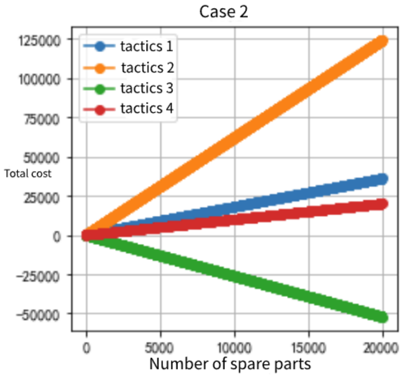

Figure 6. Total Cost for Scenario 2

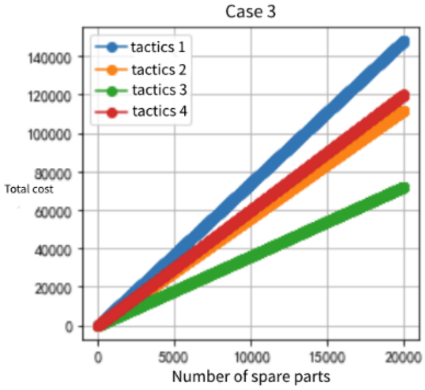

Figure 7. Total Cost for Scenario 3

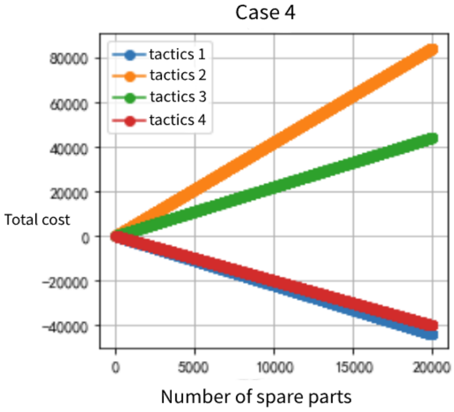

Figure 8. Total Cost for Scenario 4

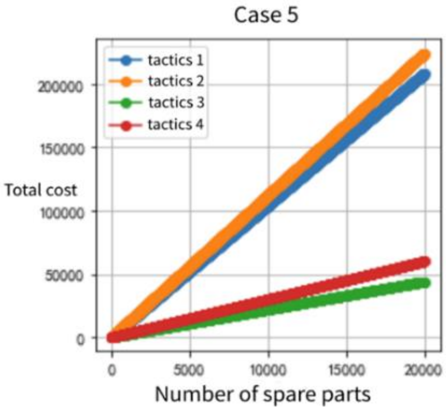

Figure 9. Total Cost for Scenario 5

Figure 10. Total Cost for Scenario 6

Figures 4 to 10 show that strategy 1 has the highest total cost, which increases rapidly with the number of spare parts. This suggests higher detection and disassembly costs in strategy 1. Strategy 2 has the lowest total cost, indicating it may be a better cost control strategy. The total cost of strategy 3 is slightly higher than that of strategy 2 but lower than that of strategy 4, while strategy 4's total cost is slightly higher than that of strategy 1. It can be speculated that strategy 2 saves costs through efficient testing or disassembly processes. Consider six different situations: Analyze steps (3) and (4) again, encode the strategy, and calculate the minimum cost for the 16 decision-making scenarios as shown in Table 4.

Table 4. Decision Results

Scenario	Strategy Code	Inspection of Part 1	Inspection of Part 2	Inspection Cost	Inspection Cost	Disassembly Cost
1	Strategy 16	No	No	No	No	9.0
2	Strategy 15	No	No	No	Yes	21.0
3	Strategy 14	No	No	Yes	No	13.0
4	Strategy 1	yes	Yes	Yes	Yes	8.0
5	Strategy 12	No	Yes	Yes	No	10.0
6	Strategy 16	no	No	No	No	5.5

It is essential to examine the correlation between spare parts quantity and profit, revealing a nonlinear relationship. To explore this further, we varied the number of spare parts from 5 to 11,995 in increments of 10, aiming to determine if spare parts quantity influences optimal decision-making. The resulting data is summarized in the table below, with Image 11 illustrating the findings

Table 5. Strategy 16 Cost Control Analysis

Scenario	Strategy Code	Inspection of Part 1	Inspection of Part 2	Inspection Cost	Disassembly Cost
5	Strategy16	9.0	11995	Strategy 16	21519
15	Strategy16	27.0	11965	Strategy 16	21537
25	Strategy16	45.0	11975	Strategy 16	21555
35	Strategy16	63.0	11985	Strategy 16	21573
45	Strategy16	81.0	11995	Strategy 16	21591

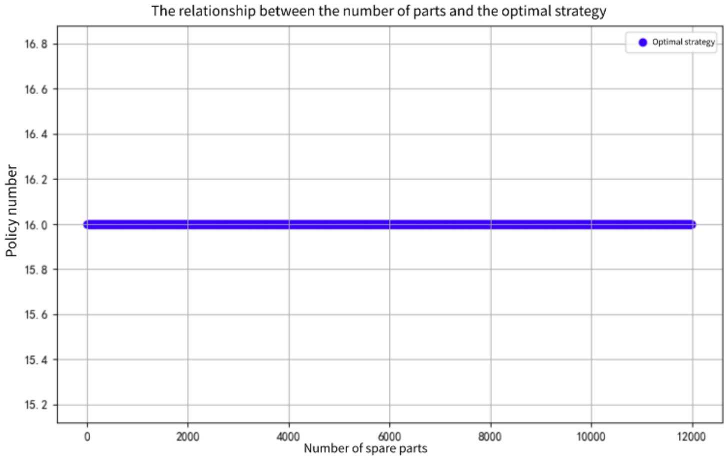

Figure 11. Number of Parts and Decision for Scenario 1

Observing the table, it is found that every time the number of spare parts increases by 10 pieces, the cost increases by 18 yuan and the increase in the number of spare parts does not affect the decision-making. Draw a graph showing the relationship between the number of spare parts and the optimal decision-making for other situations:

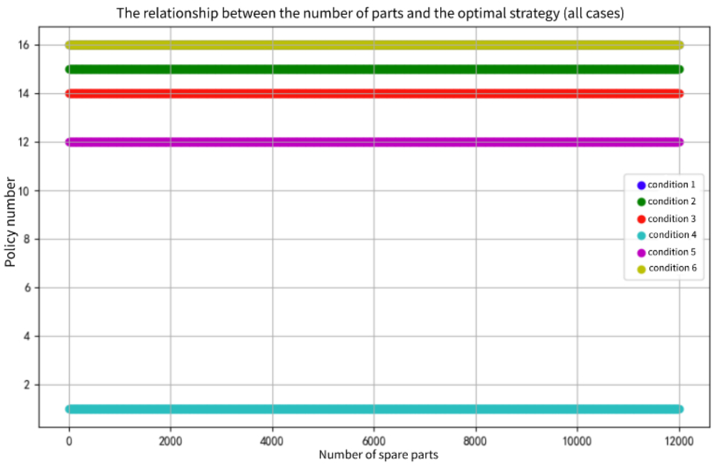

Figure 12. Decision on the Number of Parts for Remaining Scenarios

Observing Figure 12, it is found that the increase in the number of spare parts does not affect the optimal decision-making.

4. Results

Using the sequential sampling method, the rejection hypothesis $P \le p_0, H_1 \ge p_1$, was obtained, and the hypothesis of $p = 1.5$ was assumed. For each sample, 1000 simulations were conducted, and the number of times when the rejection hypothesis was rejected in two cases was 1137 and 2095 respectively. The number of times when the acceptance hypothesis was accepted was 51 and 106. When planning production for data, it was found that the cost of spare parts and finished products showed a linear relationship .Through multi-stage dynamic planning model, the minimum total cost of DP matrix solution was 13068.99, and the conclusion was drawn that all unqualified products should be disassembled after testing half-finished products and finished products.

5. Conclusion

Divide the production process into multiple stages and apply dynamic programming to accurately reflect the actual situation of the production process. Combine decision trees with machine learning models (such as random forests or neural networks) to improve prediction accuracy and the ability to handle complex data relationships. Apply these methods to more complex production systems, such as multi-plant or supply chain environments, to achieve global optimization. These future research directions can not only enhance the applicability of existing models but also provide more effective solutions for production decision-making and inspection processes in the actual manufacturing environment.

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