Research on Production Process Problems Based on Dynamic Programming

Muwei Sun *, Yutong Zhu

School of Mechanical Engineering and Automation, Northeastern University, Shenyang, China, 110000

* Corresponding Author Email: youyouleo74647@163.com

Abstract. This paper addresses the quality inspection and cost optimization issues faced by an electronic product manufacturing enterprise by proposing a hybrid decision-making model based on sequential detection and dynamic programming. A truncated sequential confidence inspection model is constructed by integrating sequential sampling and likelihood ratio test methods. Under the condition of a nominal defective rate of 10%, the model enables dynamic adjustment of the sampling volume. When the actual defective rate is 20%, the model can reject an entire batch of spare parts with a relatively small number of samples. When the defective rate is close to the nominal value, it can reduce the number of samples to avoid excessive inspection. Meanwhile, a multi-stage dynamic programming model is established to minimize costs, and the optimal decision-making schemes for six typical production scenarios are derived and verified. This model provides decision-making support for enterprises that balances quality control and cost-effectiveness.

Keywords: Truncated Sequential Sampling, Likelihood Ratio Test (LRT), Dynamic Programming, Total Cost Minimization, Production Decision Optimization.

1. Introduction

In recent years, the escalating demands for quality control and cost optimization in manufacturing have propelled the design of efficient sampling inspection schemes and the optimization of multi-stage production decisions to the forefront of academic and industrial research. While scholars worldwide have conducted extensive investigations in related domains, several theoretical gaps persist, particularly in dynamically integrating inspection strategies with cost-driven decision-making under uncertain production environments.

Existing research in sampling inspection exhibits notable limitations. Zhang et al. (2011) [1] proposed a generalized attribute-based sequential sampling method, enhancing the efficiency of traditional sequential sampling through asymmetric testing intervals. However, their framework overlooked the cost implications of dynamically adjusting sample sizes, a critical factor in practical manufacturing. Zhang et al. (2006) [2] analyzed sampling decisions in supply chain quality management from a game-theoretic perspective but assumed a fixed defect rate, which fails to account for the dynamic uncertainties inherent in real-world production processes. Hao et al. (2024) [3] leveraged Bayesian inference to optimize non-probabilistic sampling estimation, yet their approach lacked integration with sequential inspection frameworks, precluding real-time updates of defect rates based on cumulative data. Moreover, most studies remain theoretically oriented, with insufficient systematic optimization of inspection costs and decision pathways across complex, multi-stage production scenarios.

Against this backdrop, sequential sampling emerges as a pivotal methodology for manufacturers to assess supplier component defect rates under resource constraints (e.g., time, budget). As a progressive inspection technique, sequential sampling allows adaptive adjustment of subsequent sampling strategies based on prior batch results, enabling efficient allocation of inspection efforts [4-5]. The likelihood ratio test (LRT) serves as a statistically rigorous criterion in this process, providing a quantitative basis for accepting or rejecting product batches by evaluating whether cumulative inspection results meet predefined quality thresholds [6-7]. Truncation-introducing a maximum sample size limit in sequential sampling-further ensures practical feasibility by preventing indefinite sampling, compelling a final decision (acceptance or rejection) once the upper bound is reached [8].

To address these research gaps, this study proposes a hybrid methodology integrating dynamic sampling with intelligent optimization. First, a truncated sequential credibility inspection model is designed to dynamically adjust sampling strategies for component defect rate verification, minimizing inspection costs while maintaining statistical rigor. Second, a multi-stage dynamic programming (DP) model is developed to systematically optimize production decisions across inspection, assembly, and defect handling stages, quantifying inter-stage cost dependencies to derive globally optimal solutions. The core innovations lie in: 1) the synergistic integration of truncation design and LRT to achieve adaptive sample size reduction under specified confidence levels; and 2) the formulation of DP-based state transition equations to reconcile inspection, assembly, and disassembly costs, enabling holistic optimization of production workflows. By bridging theoretical methodology with practical cost considerations, this research provides a data-driven decision framework for balancing quality assurance and operational efficiency, particularly suited to high-mix, low-volume manufacturing environments.

2. Research on Design of Product Defect Rate Inspection Methodology

2.1. Design of Inspection Methodology

(1) Basic Assumptions and Parameter Definition

The defect rate inspection fundamentally constitutes a repetitive Bernoulli trial process, where the sample population can be modeled as a binomial distribution. Sequential sampling is implemented by iteratively inspecting one component per trial until predefined stopping criteria are satisfied.

Let the nominal defect rate (specified threshold) be p_0 and the true population defect rate be p. The hypothesis framework is formalized as null hypothesis $H_0: p \le p_0$, alternative hypothesis $H_1: p > p_0$.

Following each sequential sampling iteration, the likelihood ratio λ is computed and utilized to determine whether to accept H0 or continue sampling.

(2) Setting Sampling Termination Criteria

Based on statistical principles, two thresholds can be established to determine sampling termination: the upper stopping threshold $S_A = \frac{1-\beta}{\alpha}$ (for rejecting H_0)and the lower stopping threshold $S_B = \frac{\beta}{1-\alpha}$ (for accepting H_1).

α denotes the probability of committing a Type I error (incorrectly rejecting a true H_1). β denotes the probability of committing a Type II error (incorrectly accepting a false H_1).

(3) Sequential Sampling and Decision-Making

Perform sequential sampling on the components. After each sampling an inspection, calculate the likelihood ratio λ.

Under the binomial distribution assumption, suppose n components have been sampled, with x defective items. The sample defect rate is estimated as $\hat{p} = \frac{x}{n}$.

Under the null hypothesis H_0, the nominal defect rate is $p_0 = 10\%$. If the defect rate under H_0 is significantly greater than 10%, it can be hypothesized as p_1.

According to the definition of the likelihood function, it follows that

$$L(H) = p^x (1-p)^{n-x} \tag{1}$$

Thus, the likelihood ratio λ is:

$$\lambda = \frac{p_0^x (1-p_0)^{n-x}}{p_1^x (1-p_1)^{n-x}} \tag{2}$$

If $\lambda > S_A$, reject H0 and accept H1; If $\lambda < S_B$, accept H_0 and reject H_1; If $S_A < \lambda < S_B$, continue sampling.

2.2. Model Solution

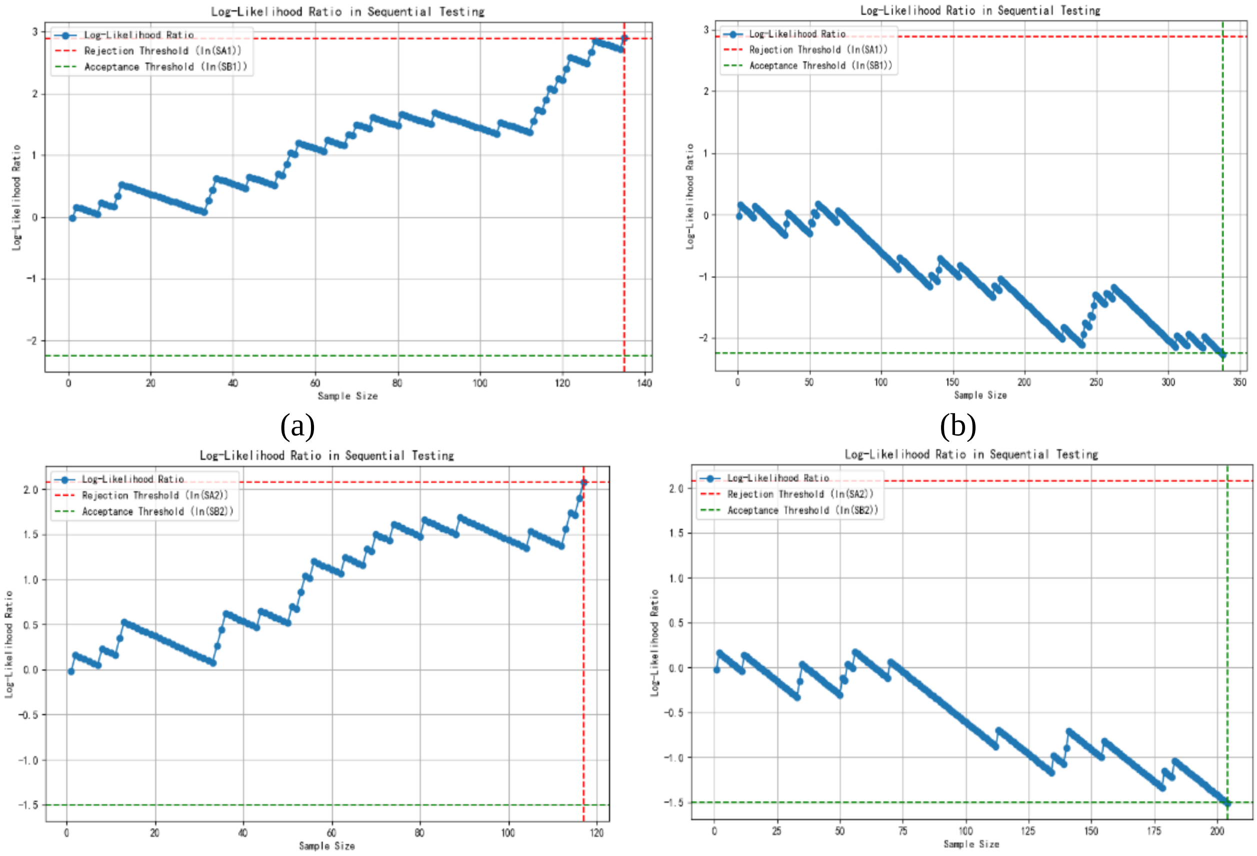
In practical production processes, manufacturers must make acceptance or rejection decisions under different confidence levels. This study evaluates the proposed model under two specific confidence levels: 95% (Case 1) and 90% (Case 2). Based on these confidence levels, the corresponding Type I error probability α is calculated. Empirical statistical knowledge suggests that α and $1-\beta$ exhibit an approximate inverse relationship. Therefore, during the solution process, the value of β is selected according to α.

For Case 1 (95% confidence level), we obtain $\alpha = 0.05$ and $\beta = 0.1$, with the upper stopping threshold $S_{A1} = 18$ and the lower stopping threshold $S_{B1} \approx 0.105$; For Case 2 (90% confidence level), we have $\alpha = 0.1$ and $\beta = 0.2$, with the upper stopping threshold $S_{B1} \approx 0.222$.

Based on the stopping boundaries (S_A, S_B) under both scenarios, sequential sampling is performed on the component population. The likelihood ratio λ is calculated, and corresponding decisions are made according to the model rules.

2.3. Analysis of Model Results

To further validate the model's rationality, two sample categories were selected for visualization on the Python platform: one with defect rates significantly exceeding the nominal value and the other approximating the nominal value, as illustrated in Figure 1.



(a) (b) (c) (d)

Figure 1. Variation of Log-Likelihood Ratio in Sequential Analysis

Table 1. Description of Likelihood Ratio Logarithmic Value Changes

	Case 1 (95%)		Case 2 (90%)	
Defect Rate	0.2	0.08	0.2	0.08
Stopping Sample Size	135	338	117	204
Logarithm of Likelihood Ratio	2.905≥ln (S_{A1})	-2.271≤ln (S_{B1})	2.081≥ln (S_{A2})	-1.513≤ln (S_{B2})
Corresponding Figure	Reject H_0	Accept H_0	Reject H_0	Accept H_0
Corresponding Figure	Figure 1a	Figure 1b	Figure 1c	Figure 1d

To validate the model's rationality, this study visualizes log-likelihood ratio trends for two sample types-20% (high defect rate) and 8% (near-nominal defect rate)-under 95% and 90% confidence levels via Python (Figure 1). Four subplots illustrate the dynamic decision mechanism.

For 20% defect rates (subplots (a, c)), the log-likelihood ratio rises rapidly, exceeding the upper threshold to trigger rejection: 135 samples (95% confidence) and 117 samples (90% confidence), both fewer than fixed-sample methods, demonstrating efficient risk screening and adaptability to confidence requirements.

At 8% defect rates (subplots (b, d)), the ratio stabilizes around zero before declining to the lower threshold for acceptance: 338 samples (95%) and 204 samples (90%), minimizing over-inspection compared to fixed sampling. Confidence-level differences validate the inverse efficiency-correlation, guiding strategy adjustments.

Table 1 and subplots show the model uses likelihood ratio dynamics to reduce inspections for deviating defects and prevent waste near nominal levels, balancing quality and cost effectively.

3. Research on Cost Optimization Decision Model

3.1. Establishment of Minimum-Cost Decision Model

Following the requirements for decision model construction, the objective function is defined to maximize corporate profit [9].

$$\max \pi = v - c \tag{3}$$

(1) Stage-Specific Cost Analysis

For Stage 1, costs include material costs and inspection costs.

The manufacturer decides whether to inspect components. If inspected, the process incurs a unit inspection cost c_i per component. Qualified components proceed to assembly, while defective ones are removed. The cost of defective components is calculated as $p_i \cdot c_{pi}$, where p_i is the defect rate of component i, and c_{pi} denotes the unit purchase price of component i.

For Stage 2, costs include assembly costs and inspection costs, while revenue derives from the market selling price of finished products.

The enterprise incurs a unit assembly cost c_s for assembling components into finished products. A decision is made on whether to inspect the assembled products. If inspected, a unit inspection cost c_c is incurred. The revenue from qualified products is calculated as:

$$v - \sum_{i=1}^{2} c_{pi} - c_s \tag{4}$$

Defective products detected during inspection proceed to the next stage's decision-making, while non-inspected finished products are entirely released into the market. To address the complexity and consistency of model variables, the revenue from finished products is converted into a negative cost $-v$, and the product manufacturing cost c_t is defined as:

$$c_t = \sum_{i=1}^{2} c_{pi} + c_s - v \tag{5}$$

For Stage 3, costs include disassembly costs, opportunity costs of discarding finished products, and reuse costs of disassembled components.

The enterprise replaces non-inspected defective products, incurring a unit replacement cost c_d. For defective products detected post-inspection and returned defective products, a disassembly decision is made:

If disassembly is chosen, a unit disassembly cost c_r is incurred. Disassembled components are then reused, with a unit reuse cost c_u .If disassembly is declined, an opportunity cost c_0 is incurred due to product discard.

Integrating the Stage 3 analysis, the total cost c_{total} of finished products can be expressed as:

$$c_{total} = c_i + c_c + c_t + c_r + c_u + c_d \tag{6}$$

(2) Objective Function Formulation

After converting the revenue from finished products into a negative cost $-v$, the objective function is transformed into a cost minimization function $\min c$. Integrating the cost analyses across all stages outlined above, the cost minimization function $\min c$ is defined by the following relationship:

$$\min c = c_{total} \tag{7}$$

During the model establishment phase, multiple interdependent stages (e.g., component inspection, finished product assembly, product inspection, and defective product handling) and state transitions (e.g., component qualification status influencing subsequent decisions) are involved. These stages exhibit dependency relationships, where decision outcomes from prior stages directly affect the costs and benefits of subsequent stages.

Dynamic programming is well-suited for such problems due to its ability to handle overlapping subproblems and exploit optimal substructure properties [10]. By formulating state transition equations and preserving intermediate computational results, the method eliminates redundant calculations, thereby efficiently identifying the global optimal solution [11].

3.2. Model Solution

(1) Stage Identification in Dynamic Programming Process

The stages in the dynamic programming process align with the three production stages established in the model (Section 3.1).

(2) Decision Variables and Allowable Decision Sets

For Stage 1, the decision variable d_i represents the inspection decision for component i, where:

$$d_i = \begin{cases} 0, & do\,not\,inspect\,component\,i \\ 1, & inspect\,component\,i \end{cases} (i = 1, 2) \tag{8}$$

For Stage 2, the decision variable d_3 represents the inspection decision for finished products:

$$d_3 = \begin{cases} 0, & do\,not\,inspect\ finished\ products \\ 1, & inspect\ finished\ products \end{cases} \tag{9}$$

For Stage 3, the decision variable d_4 represents the disassembly decision for defective finished products:

$$d_4 = \begin{cases} 0, & do\,not\,disassemble\,defective\ finished\ products \\ 1, & disassemble\,defective\ finished\ products \end{cases} \tag{10}$$

Allowable decision set:

$$D = \{d_1, d_2, d_3, d_4\} \tag{11}$$

(3) Formulation of State Transition Equation

The state variable s_k represents the cumulative set of all decisions made from the initial stage to the current stage. The state transition equation is formulated as follows:

$$s_k = \begin{cases} d_4, & k = 3 \\ d_3, d_4, & k = 2 \\ d_1, d_2, d_3, d_4, & k = 1 \end{cases} \tag{12}$$

(4) Determination of Optimal Value Function

The optimal value function $V_i(s_i)$, representing the cumulative cost from the initial stage to the current stage, satisfies the following recursive relationship. The dynamic programming approach is applied in a backward induction manner to solve for the minimum total cost from Stage 3 to Stage1.

For Stage 3, disassembled products comprise two categories: Non-inspected replaced products (defective items returned without prior inspection). Inspected defective products (identified through quality checks in Stage 2).

When $d_4 = 0$, defective products are not disassembled and are directly discarded. When $d_4 = 1$, the enterprise incurs the reuse cost $d_4 \cdot c_u$ for inspected defective products, while bearing the replacement cost $(1 - d_3) \cdot c_d$ for non-inspected defective products and recovering their material cost $(1 - d_3) d_4 \cdot c_m$.

Here, the reuse cost cu comprises the disassembly cost cr and the recycled material cost $V_1(d_1, d_2)$, mathematically expressed as:

$$c_u = c_r - V_1(d_1, d_2) \tag{13}$$

Therefore, the mathematical expression for $V_3(s_3)$ is:

$$V_3(s_3) = d_4 \cdot c_u + (1 - d_3) \cdot [c_d + c_t - d_4 \cdot c_m] \tag{14}$$

For Stage 2, when $d_3 = 0$ (no inspection of finished products), $V_2(s_2)$ equals the manufacturing cost c_t; when $d_3 = 0$ (inspection performed), $V_2(s_2)$ includes the unit inspection cost c_c, the expected cost $E_r[p_{fail}]$ for defective products entering Stage 3, and the expected cost $E_o[p_{fail}]$ for qualified products in Stage 2. The expected cost for defective products entering Stage 3 is defined as:

$$E_r[p_{fail}] = p_{fail} \cdot V_3(s_3) \tag{15}$$

p_{fail} denotes the actual defect rate of finished products.

$$p_{fail} = 1 - (1 - d_1 \cdot p_1) \cdot (1 - d_2 \cdot p_2) \cdot (1 - p_s) \tag{16}$$

The expected cost $E_o[p_{fail}]$ for conforming products in Stage 2 is defined as follows:

$$E_o[p_{fail}] = (1 - p_{fail}) c_t \tag{17}$$

Therefore, the mathematical expression for $V_2(s_2)$ can be formulated as:

$$\begin{aligned} V_2(s_2) &= d_3 \cdot \{c_c + E_r[p_{fail}] + E_o[p_{fail}]\} + (1-d_3) \cdot c_t \\ &= d_3 \cdot [c_c + p_{fail} \cdot V_3(s_3) + (1-p_{fail}) \cdot c_t] + (1-d_3) \cdot c_t \end{aligned} \tag{18}$$

Integrating the recursive processes across all three stages, the optimal value function for the initial stage is determined as follows:

$$V_1(s_1) = \sum_{i=1}^{2} d_i \cdot [c_i + p_i \cdot c_{pi} + (1-p_i) \cdot V_2(s_2)] + (1-d_i) \cdot V_2(s_2) \tag{19}$$

Integrating the recursive processes across all three stages, the optimal value function for the initial stage is determined as follows:

$$\min_{s_1} V_1(s_1) \tag{20}$$

Based on the established optimal value functions, this study derives optimal decisions for six typical production scenarios encountered by the enterprise in component manufacturing, as detailed in Table 2. Given the objective of minimizing total costs, a negative optimal total cost indicates corporate profitability, whereas a positive value signifies financial losses.

Table 2. Six Typical Production Scenarios Encountered by the Enterprise

Scenario	Component 1			Component 2			Finished Products				Defective Finished Products	
	Defect Rate	Unit Purchase Price	Inspection Cost	Defect Rate	Unit Purchase Price	Inspection Cost	Defect Rate	Unit Purchase Price	Inspection Cost	Market Selling Price	Replacement Loss	Disassembly Cost
1	10%	4	2	10%	18	3	10%	6	3	56	6	5
2	20%	4	2	20%	18	3	20%	6	3	56	6	5
3	10%	4	2	10%	18	3	10%	6	3	56	30	5
4	20%	4	1	20%	18	1	20%	6	2	56	30	5
5	10%	4	8	20%	18	1	10%	6	2	56	10	5
6	5%	4	2	5%	18	3	5%	6	3	56	10	40

Using a reference batch of 100 finished products for each scenario, the detailed decision outcomes are summarized in Table 3.

Table 3. Description of Optimal Decisions for Each Scenario

Scenario	Profit Amount	Stage 1	Stage 2	Stage 3
1	2060.0	Do not inspect Component 1 and Component 2	Do not inspect finished products	Do not disassemble defective products
2	1320.0	Do not inspect Component 1 and Component 2	Do not inspect finished products	Do not disassemble defective products
3	1440.0	Inspect Component 1 and Component 2	Do not inspect finished products	Do not disassemble defective products
4	980.0	Inspect Component 1 and Component 2	Do not inspect finished products	Disassemble defective products
5	1940.0	Do not inspect Component 1; inspect Component 2	Do not inspect finished products	Do not disassemble defective products
6	2430.0	Do not inspect Component 1 and Component 2	Do not inspect finished products	Do not disassemble defective products

Table 3 reveals varying optimal decisions in different production scenarios. In most cases (e.g., Scenarios 1, 2, 6), high inspection costs or near-nominal defect rates lead to no inspection in Stages 1 and 2, controlling costs for profit. When defect rates rise (Scenarios 3, 4) or inspection costs are low, Stage 1 inspects components. In Scenario 4, high replacement losses prompt Stage 3 to disassemble defective products. The model balances stage-wise costs via dynamic programming, aligning decisions with profit goals and validating multi-stage optimization.

3.3. Model Evaluation

To further validate whether the optimal solutions derived by the minimum-cost decision model for the six typical scenarios align with practical feasibility, we perform an exhaustive enumeration of all 16 decision possibilities for each scenario, resulting in a total of 96 possibilities.

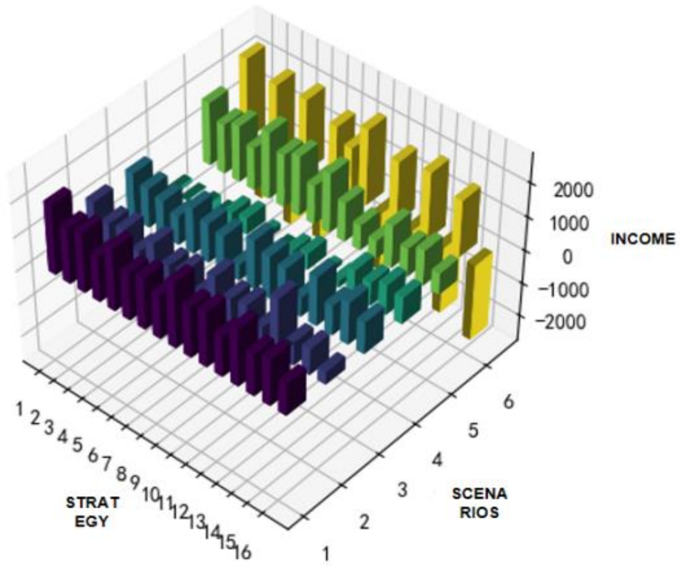


Figure 2. Profit Amount Comparison Chart for All Decision Scenarios

The profit amounts for all possible scenarios are visualized as shown in Figure 2. After exhaustive enumeration, the six optimal decisions corresponding to minimum cost and maximum revenue for the six typical scenarios align perfectly with the optimal solutions derived by the minimum-cost decision model for the cases in Table 2. Based on this, it can be concluded that the model provides enterprises with optimal decision-making solutions from a minimum-cost perspective.

4. Conclusion

This study proposes a hybrid decision-making model integrating truncated sequential sampling with dynamic programming to address quality inspection and cost optimization challenges in electronics manufacturing.

The truncated sequential credibility inspection model dynamically adjusts sampling strategies based on likelihood ratio tests (LRT). At a 95% confidence level, it efficiently rejects batches with a 20% defect rate using fewer samples than fixed sampling methods and reduces unnecessary inspections when the defect rate is close to the nominal 10%, significantly lowering inspection costs while maintaining statistical rigor. The multi-stage dynamic programming model, solving via backward induction, derives optimal decisions for six typical production scenarios, quantifying cost dependencies across inspection, assembly, and disassembly stages to achieve global cost minimization. The core innovation lies in the synergistic integration of truncation design, LRT, and dynamic programming, forming a quantifiable and reusable decision framework.

Looking ahead, this model can be applied to high-mix, low-volume manufacturing, supply chain quality management, and intelligent production systems, offering adaptive solutions to balance quality control and profitability in dynamic industrial environments.

References

[1] Zhang Zhihua, Liu Haitao. Generalized counting sequential sampling test [J]. Journal of Naval Engineering University, 2011, 23 (06): 44 – 48.

[2] Zhang Bin, Huazhong Sheng. Non-cooperative game analysis of sampling test decision in supply chain quality management [J]. China Management Science, 2006, (03): 27 - 31.

[3] Hao Yiwei, Liu Xiaoyu, Jin Yongjin. The use of prior information in non-probability sampling estimation-based on Bayesian model estimation perspective [J]. 2022, (05): 86 - 96.

[4] Chen Huijuan, Hu Sigui, Li Qiude, et. al. Truncated sequential optimal test under no-replacement test [J]. Application of probability and statistics, 2025, 41 (01): 65 - 82.

[5] Tian Zixuan, Xie Xiaoyue. Big data sequential test method and its application [J]. Statistics and information forum, 2024, 39 (09): 13 - 22.

[6] Wang Yongjuan, Yao Yan. Fan Yingbing. Case study of experimental teaching of Probability Theory and Mathematical Statistics course for big data major [J]. Heilongjiang Education (Theory and Practice), 2025, (03): 76 - 78.

[7] Shao Lei, He Yangchao, Zhao Jin. Target maneuver detection method based on likelihood ratio test [J]. Aviation weapons, 2024, 31 (06): 64 - 69.

[8] Li Taixin, Jin Xihan. Research on enterprise production decision optimization based on dynamic programming and sequential probability ratio test [J]. Market Outlook, 2024, (24): 7 - 9.

[9] Lv Zhigang, Li Ye, Wang Hongxi, et al. Review of structural learning of Bayesian networks [J]. Journal of Xi'an University of Technology, 2021, 41 (01): 1-17.

[10] Hu Zhonghua, Ge Chongwei, Xu Hailong. Research on a dynamic area planning algorithm based on production planning [J]. Equipment manufacturing technology, 2025. (01): 39 - 43.

[11] Li Yuli, Hu Hongchang. Sequential Lq likelihood ratio test [J]. Mathematics Yearbook Series A (Chinese Version), 2023, 44 (04): 363 - 382.